

Final

Please complete the following problems. Be sure to ask me if you have any questions or anything is unclear. Partial credit will be given, so **be sure to show all of your work**. If you use your calculator, write the information you input into the calculator, or the command you used.

Thanks for all of your work this summer. Good luck this fall!

1. (10 pts) The length of time needed to fall asleep at night depends on what happened the night before. A random sample of 38 college students was kept awake all night and the next day (24 hours total). The average time for this group to go to sleep the next night was $\bar{x} = 2.5$ minutes, with a standard deviation of $\sigma = 0.7$ minutes.

- a) (4 pts) Construct a 90% confidence interval for μ , the true average length of time for college students to fall asleep under these conditions.

[Calculator] STAT → TESTS → 7:ZInterval gives an interval of (2.3132 , 2.6868)

[By Hand]

$$\bar{x} \pm Z^* \frac{\sigma}{\sqrt{n}} = 2.5 \pm 1.645 \frac{0.7}{\sqrt{38}} = 2.5 \pm 0.1868 = (2.3132 , 2.6868)$$

Our 90% confidence interval is (2.3132 , 2.6868).

- b) (3 pts) What would happen to the confidence interval if we had sampled 50 students instead of 38 of them?

If we sampled 50 students instead of 38 of them, the n would be larger, and so the confidence interval would get narrower.

- c) (3 pts) What would happen to the confidence interval if we lowered our confidence level from 90% to 80%?

Lowering the confidence level from 90% to 80% will decrease our Z^* value, and our confidence interval will get narrower.

2. (6 pts) A high-tech company wants to estimate the mean number of years of college education its employees have completed. A good estimate of the standard deviation for the number of years of college is $\sigma = 1.6$. How large a sample needs to be taken to estimate the mean number of college years to within 0.5 years with 98% confidence?

We need our margin of error, m , to be 0.5. Using the sample size formula,

$$n = \left(\frac{Z^* \sigma}{m} \right)^2 = \left(\frac{2.326 \times 1.6}{0.5} \right)^2 = 7.4432^2 = 55.401.$$

We always round up for sample size calculations, so we'd need to sample 56 employees.

3. (6 pts) One of the two assumptions about our data that we must have to perform a one-sample t-test procedure is that the data come from a simple random sample.

- a) (3 pts) What is the other assumption we must make?

We must also assume that the data are normally distributed.

- b) (3 pts) How would you go about verifying if your data satisfy the second assumption?

There are several ways to verify this. We could make a stemplot or histogram and look for a normal shape to the distribution, or we could try a normal probability plot and see if the plotted data lie on a straight line.

4. (12 pts) A researcher is interested in the health of Beluga Whales. One agency contends that the mean thickness of Beluga Whale blubber is 4.3 inches. Our researcher feels that the mean thickness is less than that, and wishes to use a hypothesis test to prove it. He measures the thickness of the whales' blubber at a specific location. He measures 15 whales, and records the following data (in inches):

4.4	3.2	4.3	3.9	4.1
4.4	4.0	4.5	3.6	4.6
4.4	4.3	4.0	4.8	3.9

Assume here that $\sigma = 0.4$ inches.

- a) (3 pts) Write down the appropriate null and alternative hypotheses for the researcher's test.

Let μ be the mean thickness of Beluga Whale blubber.

We wish to test:

$$H_0: \mu = 4.3$$

$$H_a: \mu < 4.3$$

- b) (2 pts) Would you use a t-test or a z-test here? Explain your reasoning.

Since we are given the standard deviation $\sigma = 0.4$, we should use a z-test.

- c) (2 pts) What is the value of the test statistic you calculated?

[Calculator] STAT \rightarrow TESTS \rightarrow 1:Z-Test gives a test statistic of $Z = -1.356$.

[by hand] Using the calculator, we have $\bar{x} = 4.16$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{4.16 - 4.3}{\frac{0.4}{\sqrt{15}}} = -1.356. \text{ The test statistic is } Z = -1.356.$$

- d) (2 pts) What is the p-value for this hypothesis test?

[Calculator] STAT \rightarrow TESTS \rightarrow 1:Z-Test gives a p-value of 0.0876.

[by hand] Our p-value is $P(Z < -1.36)$. From Table A, we get 0.0869.

- e) (3 pts) What is your conclusion? Use $\alpha = 0.05$.

Since our p-value is greater than 0.05, we do not reject H_0 . We conclude that the mean thickness of Beluga Whale blubber is 4.3 inches.

5. (12 pts) A company with a large fleet of cars hopes to keep gasoline costs down, and sets a goal of attaining a fleet average of 26 miles per gallon.

To see if the goal is being met, they check the gasoline usage for 30 company trips chosen at random, finding a mean of $\bar{x} = 25.02$ mpg, and a sample standard deviation of $s = 2.83$ mpg.

Conduct a hypothesis test to see if the company fleet is averaging less than 26 miles per gallon. Be sure to state the null and alternative hypotheses, present your test statistic and p-value, and draw the appropriate conclusion. Use $\alpha = 0.05$.

We do not know σ here. We calculated s from our data, so we will use a 1-sample t-test. Let μ be the mean mileage per trip for the fleet. We wish to test:

$$H_0: \mu = 26 \quad (3 \text{ pts})$$
$$\text{vs } H_a: \mu < 26$$

(3 pts) Next, we need a t-test statistic.

[Calculator] STAT \rightarrow TESTS \rightarrow 2:T-Test gives a test statistic of $t = -1.897$.

[by hand]

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{25.02 - 26}{2.83/\sqrt{30}} = -1.897. \quad \text{The test statistic is } t = -1.897.$$

(3 pts) Next, we need a p-value.

[Calculator] STAT \rightarrow TESTS \rightarrow 2:T-Test gives a p-value of 0.0339.

[by hand] Our t statistic is -1.897. Using the row of Table C corresponding to $n - 1 = 29$ degrees of freedom, we see that our t-statistic is bracketed by 1.669 and 2.045, corresponding to a p-value between 0.025 and 0.05.

We could have also found $P(T < -1.897)$ using the calculator:

$$\text{tcdf}(-1E99, -1.897, 29) = 0.0339$$

(3 pts) Finally, we need a conclusion. Since our p-value is less than $\alpha = 0.05$, we reject H_0 . We conclude that the mean trip mileage is less than 26 mph.

6. (12 pts) 8 runners were asked to run a 10-kilometer race on 2 consecutive weeks. In the first week they wore shoes from Brand A, and in the second week they wore shoes from Brand B. All of them were times in minutes as given below:

Runner	Brand A	Brand B	Difference
1	31.23	32.02	0.79
2	29.33	28.98	-0.35
3	30.50	30.63	0.13
4	32.20	32.67	0.47
5	33.08	32.95	-0.13
6	31.52	31.53	0.01
7	30.68	30.83	0.15
8	31.05	31.10	0.05

A local shoe store would like to determine if there is evidence that the mean time using Brand A is less than the mean time using Brand B.

- a) (6 pts) Which test would you use here: The one-sample Z-test, matched pairs t-test, one-sample t-test, or two-sample t-test? Explain your reasoning.

We would use the matched pairs t-test here. The 10k measurements for the Brand A group and for the Brand B group were made on the same individuals.

- b) (2 pts) State the hypotheses that the store would like to test.

Let μ be the mean difference in 10k times (Brand B - Brand A).

We wish to test: $H_0: \mu = 0$
vs $H_a: \mu > 0$

- c) (2 pts) What is the test statistic and resulting p-value?

Start by getting the time difference for each runner.

[Calculator] STAT \rightarrow TESTS \rightarrow 2:T-Test gives a test statistic of $t = 1.123$, and a p-value of 0.1491

[by hand] Using the calculator and the differences, $\bar{x} = 0.14$, $s = 0.3525$.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{0.14 - 0}{0.3525/\sqrt{8}} = 1.123. \text{ The test statistic is } t = 1.123.$$

Using the row of Table C corresponding to $n - 1 = 7$ degrees of freedom, we see that our t-statistic is bracketed by 1.119 and 1.415, corresponding to a p-value between 0.10 and 0.15.

We could have also found $P(T > 1.123)$ using the calculator:
 $\text{tcdf}(1.123, 1E99, 7) = 0.1492$

d) (2 pts) What can the shoe store conclude? Use $\alpha = 0.10$.

Since our p-value is larger than $\alpha = 0.10$, we do not reject H_0 , and conclude that there is no difference in the mean 10k time using Brand A or Brand B.

7. (12 pts) A taste testing study had 100 male and 100 female participants to investigate whether taste preferences was related to a person's gender. Both groups rated their preference on a scale of 1 to 10 (1 being 'very unpleasant' and 10 being 'very pleasant'). The mean ratings and sample standard deviations for the males and females are given below:

$$\begin{array}{llll} \text{Females:} & \bar{x}_1 = 7.0 & s_1 = 2.0 & n_1 = 100 \\ \text{Males:} & \bar{x}_2 = 6.4 & s_2 = 1.5 & n_2 = 100 \end{array}$$

Let μ_1 and μ_2 represent the mean ratings one would observe for the populations of females and males respectively.

- a) (6 pts) Calculate a 95% confidence interval for $\mu_1 - \mu_2$.

[Calculator] STAT → TESTS → 0:2-SampTInt gives an interval of (0.1068 , 1.0932)

[By Hand] We use the t^* corresponding to the smaller of $n_1 - 1 = 99$ or $n_2 - 1 = 99$ degrees of freedom. The closest we can get is row 100, with $t^* = 1.984$.

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (7 - 6.4) \pm 1.984 \sqrt{\frac{2^2}{100} + \frac{1.5^2}{100}} = 0.6 \pm 0.496 = (0.104 , 1.096)$$

Our 95% confidence interval is (0.104 , 1.096).

- b) (6 pts) Suppose in the above study, a researcher had wished to test the hypotheses

$$\begin{array}{l} H_0: \mu_1 = \mu_2 \\ \text{versus } H_a: \mu_1 \neq \mu_2 . \end{array}$$

Perform the relevant hypothesis test. What do you conclude? Use $\alpha = 0.05$.

We are using a two-sample t-test.

[Calculator] STAT → TESTS → 4:2-SampTTest gives a test statistic of $t = 2.4$, and a p-value of 0.0174

[by hand]

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{7 - 6.4}{\sqrt{\frac{2^2}{100} + \frac{1.5^2}{100}}} = 2.4 \quad \text{The test statistic is } t = 2.4.$$

Use the row of Table C corresponding to the smaller of $n_1 - 1 = 99$ or $n_2 - 1 = 99$ degrees of freedom. The closest we can get is row 100. We see that our t-statistic is bracketed by 2.364 and 2.626, corresponding

to a p-value between 0.005 and 0.01. Since this is a two-sided test, we multiply the p-values by 2 to get a p-value between 0.01 and 0.02.

We could have also found $P(T \neq 2.4)$ using the calculator:
 $\text{tcdf}(2.4, 1E99, 99) = 0.009$. For our two-sided test, we multiply by 2 to get a p-value of 0.0183.

Since our p-value is less than $\alpha = 0.05$, we reject H_0 , and conclude that the mean preferences of men and women are different.

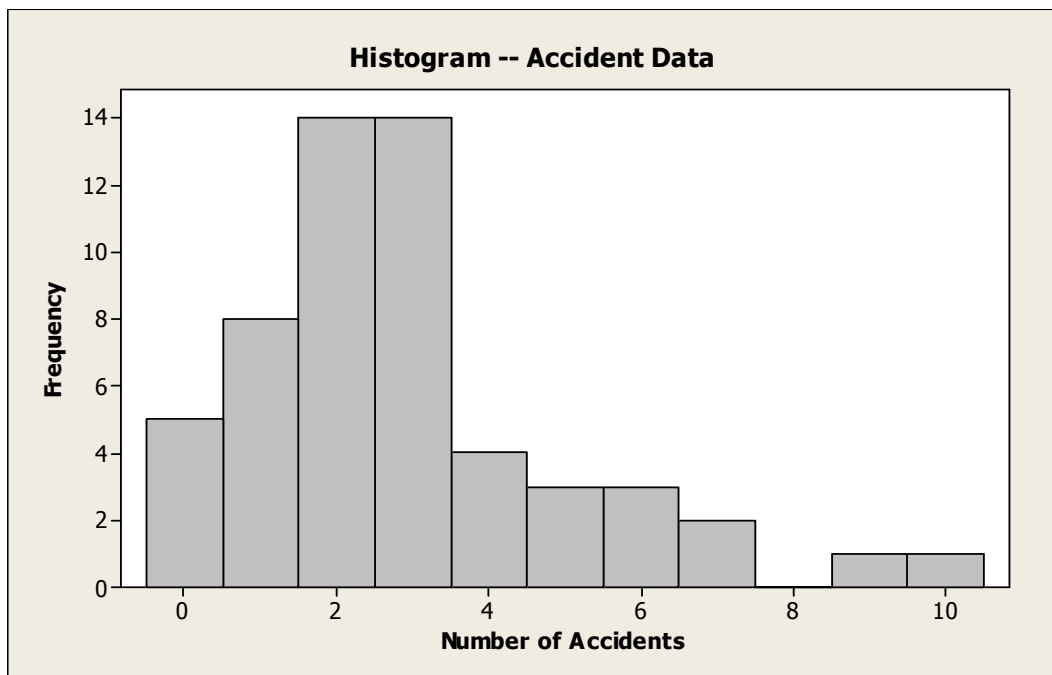
8. (12 pts) In each of the following situations, state whether or not t-procedures (a t-interval or a t-test) are appropriate.

- a) (6 pts) You are studying the number of hours in labor spent by cows. You measure the length of labor for 14 cows at the UCONN dairy barn. A stem-and-leaf plot for a set of data is given on the right. Based on this, do you think that it is appropriate to use t-procedures to create a 95% confidence interval for the mean number of hours spent in labor? Explain your reasoning.

0	33
0	5566
1	024
1	67
2	
2	6
3	1
3	
4	
4	
5	3

It is not appropriate to use t-procedures here. The sample size is quite small, and the data are skewed to the right with a possible outlier.

- b) (6 pts) You are studying the number of accidents that student at UCONN have been involved in. You select a random sample of 55 people. A histogram of the data is provided below:



Would a 90% t-interval be reasonable in this situation? Explain your reasoning.

Here, a 90% t-interval would be reasonable. Even though the data are skewed, the sample size is large.

9. (6 pts) You are studying the length of drool coming off of the jowls of St. Bernard dogs. Let μ be the mean length (in cm) of drool. You wish to test the hypotheses:

$$\begin{array}{l} H_0: \mu = 5 \\ \text{versus } H_a: \mu > 5. \end{array}$$

You plan to measure some dogs and perform the hypothesis test. In terms of this problem, describe the two possible errors that could occur when you make your conclusions.

There are two errors we could make:

Reject H_0 when it is true (Type I) - We'd conclude that the mean length of drool of St. Bernards is greater than 5 cm when it actually is 5 cm.

Accept H_0 when it is false (Type II) - We'd conclude that the mean length of drool of St. Bernards is 5 cm when it is in fact more than that.

10. (12 pts) Here are the IQ test scores for 20 seventh-grade girls in a Midwest school district:

114	100	104	89	102	91	114	114	103	105
108	130	120	132	111	128	118	119	86	72

In addition, here are the IQ test scores for 22 seventh-grade boys in the same school district:

111	107	100	107	115	111	97	112	104	106	113
109	113	128	128	118	113	124	127	136	106	123

Use these data to test if there is a difference in the mean IQ scores for boys and girls in this school district. Be sure to state the null and alternative hypotheses, present your test statistic and p-value, and draw the appropriate conclusion. Use $\alpha = 0.05$.

We are using a two-sample t-test. Let μ_1 be the mean IQ score for girls and μ_2 be the mean IQ score for boys. We wish to test:

$$\begin{array}{l} H_0: \mu_1 = \mu_2 \quad (3 \text{ pts}) \\ \text{vs } H_a: \mu_1 \neq \mu_2 \end{array}$$

(3 pts) Next, we need a t-test statistic.

[Calculator] STAT \rightarrow TESTS \rightarrow 4:2-SampTTest gives a test statistic of
 $t = -1.479$

[by hand] From our data, we have

$$\bar{x}_1 = 108 \quad s_1 = 15.4272 \quad n_1 = 20$$

$$\bar{x}_2 = 114 \quad s_2 = 10.0095 \quad n_2 = 22$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{108 - 114}{\sqrt{\frac{15.4272^2}{20} + \frac{10.0095^2}{22}}} = -1.479 \quad \text{The test statistic is } t = -1.479.$$

(3 pts) Next, we need a p-value.

[Calculator] STAT → TESTS → 4:2-SampTTest gives a p-value of 0.1489.

[by hand]

Use the row of Table C corresponding to the smaller of $n_1 - 1 = 19$ or $n_2 - 1 = 21$ degrees of freedom. We see that our t-statistic is bracketed by 1.328 and 1.729, corresponding to a p-value between 0.05 and 0.10. Since this is a two-sided test, we multiply the p-values by 2 to get a p-value between 0.10 and 0.20.

We could have also found $P(T \neq -1.479)$ using the calculator:

$P(T < -1.479) = \text{tcdf}(-1E99, -1.479, 19) = 0.0778$. For our two-sided test, we multiply by 2 to get a p-value of 0.1555.

(3 pts) Finally, we need a conclusion. Since our p-value is greater than $\alpha = 0.05$, we do not reject H_0 . We conclude that the mean IQ scores for boys and girls are the same.

BONUS: (4 pts)

A confidence interval for the mean height of 38 pine trees is (55.28 , 67.34). Suppose we know that the population standard deviation is $\sigma = 26.7$.

What is the confidence level of this confidence interval?

We have one population here, and we were given $\sigma = 26.7$. So, let's start with the formula for a Z-interval.

$\bar{x} \pm Z^* \frac{\sigma}{\sqrt{n}}$. The quantity on the right of the +/- sign is the margin of error.

We know that the margin of error for this confidence interval is

$$\frac{67.34 - 55.28}{2} = 6.03$$

Then, we can use the equation for the margin of error to solve for the Z^* statistic:

$$m = Z^* \frac{\sigma}{\sqrt{n}}$$

$$6.03 = Z^* \frac{26.7}{\sqrt{38}}$$

$$Z^* = 1.392.$$

From Table A, we find that a Z^* of 1.392 has a probability of 0.9177 to the left, which leaves 0.0823 to the right. The area to the left of -1.392 is also 0.0823. This leaves the middle area of the confidence interval as $1 - 2(0.0823) = 0.8354$.

The confidence level of this confidence interval is about 84%.

