Math 17, Section 2 – Spring 2011

Homework 7 Solutions

<u>Assignment</u>
Chapter 28: 4, 6, 14
Chapter 7: 4, 6, 12

Chapter 28

28.4] Darts A student interested in improving her dart-throwing technique designs an experiment to test 4 different stances to see whether they affect her accuracy. After warming up for several minutes, she randomizes the order of the 4 stances, throws a dart at a target using each stance, and measures the distance of the dart in centimeters from the center of the bull's-eye. She replicates this procedure 10 times. After analyzing the data she reports that the *F*-ratio is 1.41.

a) What are the null and alternative hypotheses?

Let μ_i denote the mean distance from the bull's-eye for stance i = 1,2,3,4. We wish to test

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
H_A: at least one of the means differs

b) How many degrees of freedom does the treatment sum of squares have? How about the error sum of squares?

We have a total of 40 observations, or 39 df. There are four treatments, so the MS_{Tr} has 3 d.f. This leaves 36 d.f. for the error sum of squares.

c) What would you conclude?

Looking at the *F*-table, the *F*-statistic for 3 and 36 d.f. for α = 0.10 is 2.25. Since our value is less than that, the *p*-value must be greater than 0.10. We don't have evidence to reject the null hypothesis that the four stances have the same mean distances from the bull's-eye.

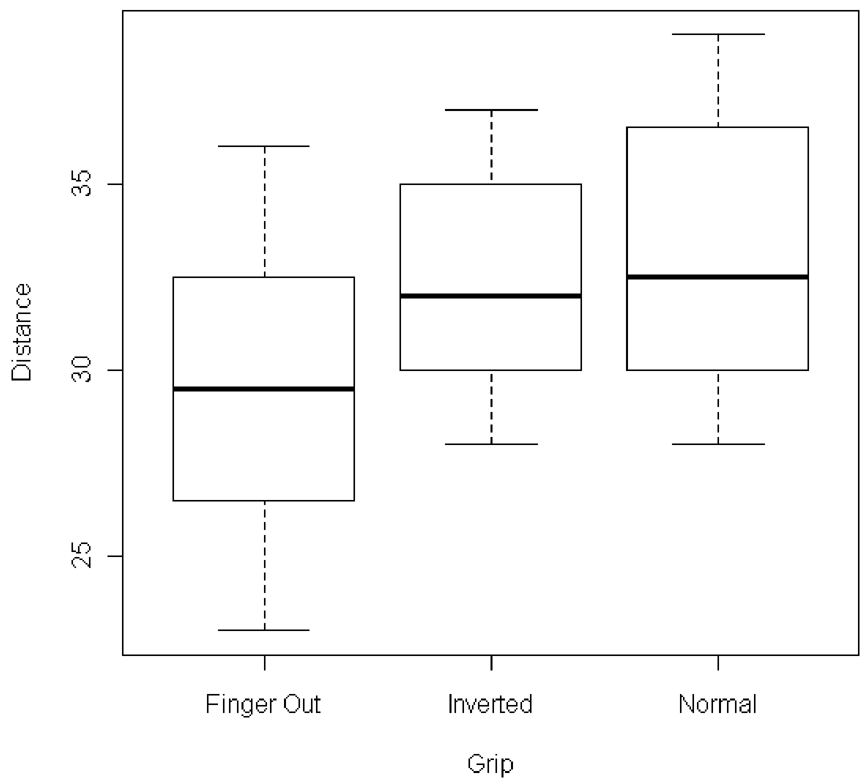
d) What else about the data would you like to see in order to check the assumptions and conditions?

We'd like to see some plots. Boxplots could help us assess the equal variances assumption, and histograms or normal probability plots could help us assess the normality condition.

e) If your conclusion in part (c) is wrong, what type of error have you made?

Our conclusion was that we failed to reject the null hypothesis. If we made an error, then, it would have to be a Type II error.

28.6] **Frisbee Throws** A student performed an experiment with three different grips to see what effect it might have on the distance of a backhanded Frisbee throw. She tried it with her normal grip, with one finger out, and with the Frisbee inverted. She measured in paces how far her throw went. The boxplots and the ANOVA table for the three grips are shown below:



Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F-ratio	P-value
Grip	2	58.58333	29.2917	2.0453	0.1543
Error	21	300.75000	14.3214		
Total	23	359.33333			

a) State the hypotheses about the grips.

Let μ_i denote the mean distance for grip i = 1,2,3. We wish to test

$H_0: \mu_1 = \mu_2 = \mu_3$
H_A: at least one of the means differs

b) Assuming that the assumptions for inference are satisfied, perform the hypothesis test and state your conclusion. Be sure to state it in terms of Frisbee grips and distance thrown.

With a p-value of 0.1543, we do not have evidence to reject the null hypothesis that the three Frisbee grips have the same mean distances.

c) Would it be appropriate to follow up this study with multiple comparisons to see which grips differ in their mean distance thrown? Explain.

No, it wouldn't be appropriate to follow up with multiple comparisons. Since we didn't reject the null hypothesis, we have no evidence of a difference in the means for the three grips, so further testing isn't needed.

28.14] **Smokestack Scrubbers** Particulate matter is a serious form of air pollution often arising from industrial production. One way to reduce the pollution is to put a filter, or scrubber, at the end of the smokestack to trap the particulates. An experiment to determine which smokestack scrubber design is best was run by placing four scrubbers of different designs on an industrial stack in random order. Each scrubber was tested 5 times. For each run, the same material was produced, and the particulate emissions coming out of the scrubber were measured (in parts per billion). A partially complete analysis of variance table of the data is shown below.

Source	Sum of Squares	DF	Mean Square	F-ratio
Treatment	81.2	3	27.067	14.06
Residual	30.8	16	1.925	
Total	112.0	19		

a) Calculate the mean square of the treatments and the mean square of the error.

b) Form the *F*-statistic by dividing the two mean squares.

c) The P-value of this *F*-statistic turns out to be 0.00000949. What does this say about the null hypothesis of equal means?

It says that we have evidence to reject the null hypothesis and conclude that at least two of the scrubbers differ in mean particulate emissions.

d) What assumptions have you made in order to answer part c?

We have assumed that the experimental runs were performed in random order, that the variances of the treatment groups are equal, and that the errors are Normal.

e) What would you like to see in order to justify the conclusions of the *F*-test?

To check the Similar Variance condition, look at side-by-side boxplots of the treatment groups to see whether they have similar spreads. To check the Nearly Normal condition, look to see if a normal probability plot of the observations is straight.

f) What is the average size of the error standard deviation in particulate emissions?

$$s_p = \sqrt{MS_E} = \sqrt{1.925} \approx 1.387 \text{ ppb}$$

This part (f) was not graded.

Chapter 7

7.4] Association

a) Time spent talking on the phone is the explanatory variable, and cost of the call is the response variable. The longer you spend talking, the more the call costs. The association is most likely positive, straight, and moderately strong, since some long distance companies charge more than others.

b) Distance from lightning is the explanatory variable, and time delay of the thunder is the response variable. The farther away you are from the strike, the longer it takes the thunder to reach your ears. The association is positive, straight, and probably fairly strong. The speed of sound is relatively constant, but will vary due to temperature, etc.

c) Distance from the streetlight is the explanatory variable, and brightness is the response variable. The further away from the light you are, the less bright it appears. The association is negative, curved, and strong. Distance and light intensity follow an inverse square relationship. Doubling the distance to the light source reduces the intensity by a factor of four.

d) There is likely very little association between the weight of the car and the age of the owner. However, some might say that older drivers tend to drive larger cars. If that is the case, there may be a positive, straight, and very weak association between weight of a car and the age of its owner.

7.6] Scatterplots.

a) #1 shows little or no association.
b) #4 shows a negative association.
c) #2 and #4 each show a straight association.
d) #3 shows a moderately strong, curved association.
e) #2 and #4 each show a very strong association, although some might classify the association as merely "strong".

7.12] Matching

a) - 0.977   b) 0.736   c) 0.951   d) - 0.021