Chapter 8 and 9 Review: A Bit More Practice

Consider n observations sampled from a distribution with pdf given by:

$$f(x|\theta) = (\theta + 1)x^{\theta}, 0 \leq x \leq 1,$$

and 0, otherwise.

a. Find the likelihood function for the n observations.
b. Identify a sufficient statistic for θ.
c. Find the MLE for θ.
d. Is the MLE minimal sufficient? Why or why not?
e. Rexpress the pdf so that it can be identified as a member of the exponential family of distributions (not exponential, just in the family).
f. Based on the distribution being in the exponential family, what other statistic can be shown to be sufficient?
g. How would you check to see if the MLE is consistent for θ?
h. Would it be appropriate to compute the relative efficiency of the estimators in c. and f. with the information you have about those estimators right now?