Math 17, Section 2 – Spring 2011

Homework 5 Solutions

<u>Assignment</u>
Chapter 19: 4, 6, 8, 10, 14, 16, 40
Chapter 20: 2, 4, 9

Chapter 19

19.4] More Conditions.

a) **Population:** The population is all customers who recently bought new cars.
Sample: The sample is the 167 people surveyed.
***p*:** The proportion of all new car buyers who are dissatisfied with the salesperson.
$\hat{p}$: The proportion of sampled new car buyers who are dissatisfied with the salesperson (3%).
Conditions: It seems reasonable to assume car buyers are independent and selected randomly. However, we find that $n\hat{p} = 167(0.03) = 5$. The sample is not large enough.

b) **Population:** The population is all college students.
Sample: The sample is the 2,883 students asked about cell phones.
***p*:** The proportion of all college students with cell phones.
$\hat{p}$: The sample proportion of students with cell phones (8.4%).
Conditions: We can possibly assume independence. One student's owing a phone shouldn't influence other, though peer pressure may change this. This is not a random sample, though. If the students at the game represent a good cross-section of the student body, then it may be okay to proceed. The other sample size requirements are satistified.

c) **Population:** The population is all potato plants in the U.S.
Sample: The sample is the 240 potato fields in Maine.
***p*:** The proportion of all potato plants in the U.S. that show the signs of blight.
$\hat{p}$: The proportion of our sample who showed blight (2.9%).
Conditions: It is unreasonable to think that signs of blight are independent. Blight is a contagious disease! Even though potato plants are randomly selected from the field in Maine, it also doesn't seem reasonable to assume that these potato plants are representative of all potato plants in the U.S. Also $n\hat{p} = 240(0.029) = 7$. The sample is not large enough. Multiple conditions are not met. We shouldn't use a CI here.

d) **Population:** The population is all employees at the company.
Sample: The sample is all employees during the specified year.
***p*:** The proportion of all employees who will have an injury in a given year.
$\hat{p}$: The sample proportion of employees who were injured this year (3.8%).
Conditions: It is reasonable to think that the injuries are independent. This sample is not random, but this year's employees are probably representative of employees in other years. The sample is large enough to use also. A CI should work here.

19.6] More Conclusions. I'll accept both (c) and (d) here. They are really similar in wording.
Part (a) has a problem: It says nothing about the confidence level.
Part (b) has a problem: We are actually 100% sure that *in this experiment* we got between 51% and 61%. That's because our experiment's results were used to compute the interval.
Part (e) has a problem: The 90% here is referring to the specific interval, rather than the *process* of getting the interval.

19.8] Confidence Intervals, again.

- **a)** True. The smaller the margin of error is, the less confidence we have in the ability of our interval to catch the population proportion.
- **b)** True. Larger samples are less variable, which translates to a smaller margin of error. We can be more precise at the same level of confidence.
- **c)** True. Smaller samples are more variable, leading us to be less confident in the ability of our interval to catch the true population proportion.
- **d)** True. The margin of error decreases as the square root of the sample size increases.

19.10] Parole.
We are 95% confident that between 56.1% and 62.5% of paroles are granted by the Nebraska Board of Parole.

19.14] Cloning, 2007.

a) The margin of error is $ME = Z^*\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(0.11)(0.89)}{1003}} = 0.0194.$

b) The pollsters are 95% confident that the true proportion of adults who approve of attempts to clone a human is within 1.9% of the estimated 11%.

c) A 90% confidence interval results in a smaller margin of error. If confidence is decreased, a smaller interval is constructed.

d) $ME = Z^*\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645\sqrt{\frac{(0.11)(0.89)}{1003}} = 0.0163$

e) Smaller samples generally produce larger intervals. Smaller samples are more variable, which increases the margin of error.

19.16] Take the offer.

a) Always remember to check the conditions first:

- Random Sample: Offers were sent to a random sample of 50,000 cardholders.
- Independence: We can assume cardholders are independent.
- 10% condition: Even though this sample is big, there are most likely more than 500,000 cardholders.
- Sample size: we have $n\hat{p} = 50000(0.02368) = 1184 \geq 10$ and $n\hat{q} = 50000(0.97632) = 48816 \geq 10$. The sample is large enough

The confidence interval is:

$$\hat{p} \pm Z^*\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.02368 \pm 1.96\sqrt{\frac{(0.02368)(0.97632)}{50000}} = 0.02368 \pm 0.00133 = (0.02235\,, 0.02501)$$

We are 95% confident that between 2.24% and 2.5% of all cardholders would register for double miles.

b) The confidence interval gives the set of plausible values with 95% confidence. Since 2% is below the interval, there is evidence that the true proportion is above 2%. The campaign should be worth the expense.

19.40] Another pilot study.

We will use $\hat{p} = 0.22$ from the pilot study as an estimate.

$$\begin{aligned} ME &= Z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \\ 0.04 &= 2.326 \sqrt{\frac{(0.22)(0.78)}{n}} \\ 0.04^2 &= 2.326^2 \frac{(0.22)(0.78)}{n} \\ n &= \frac{2.326^2(0.22)(0.78)}{0.04^2} \\ n &\approx 581 \end{aligned}$$

In order to estimate the percentage of adults with higher than normal levels of glucose in their blood to within 4% with 98% confidence, the researchers will need a sample of at least 581 adults. All decimals in the final answer must be rounded up, to the next adult.

Chapter 20

20.2] More Hypotheses.

a) Let *p* be the true proportion of high school graduates that went on to college.

$$H_0\text{: } p = 0.40$$
$$H_A\text{: } p \neq 0.40$$

b) Let *p* be the true proportion of cars needing repairs between 50k and 100k miles.

$$H_0\text{: } p = 0.20$$
$$H_A\text{: } p < 0.20$$

c) Let *p* be the true proportion of people who like the new flavor.

$$H_0\text{: } p = 0.60$$
$$H_A\text{: } p > 0.60$$

20.4] Dice.
Let *p* be the true probability that the die rolls a 6. We wish to test:

$$H_0:\ p = \frac{1}{6} \text{ (The die is fair)}$$

$$H_A:\ p > \frac{1}{6} \text{ (The die is loaded to favor 6)}$$

A *p*-value of 0.03 is found. This is quite low, and is evidence to reject the null hypothesis and conclude that the die is indeed loaded to favor 6.

A *p*-value is the probability of getting our test statistic result or something more extreme given that the null hypothesis is true. Only statement (d) correctly uses this definition.

20.9] He cheats!

a) Two losses in a row aren't convincing. There is a 25% chance of losing twice in a row, and that is not unusual.
b) If the process is fair, three losses in a row can be expected to happen about 12.5% of the time. (0.5)(0.5)(0.5) = 0.125.
c) Three losses in a row is still not a convincing occurrence. We'd expect that to happen about once every eight times we tossed a coin three times.
d) Answers may vary. Maybe 5 times would be convincing. The chances of 5 losses in a row are only 1 in 32, which seems unusual.