Math 17, Section 2 – Spring 2011

Homework 2 Solutions

<u>Assignment</u>
Chapter 6: 8, 12, 19, 27, 29, 31, 36, 40, 41, 43, 45

Chapter 6

6.8] Checkup. The grandson's *z*-score is -1.88. This means that the boy's height is 1.88 standard deviations below the mean height of 2-year-old boys. This isn't too unexpected. We might be concerned if the boy was three standard deviations below the mean, though.

6.12] Mensa. We can use the *z*-score formula here, $z = \frac{y-\mu}{\sigma}$, and solve for *y*.

$$2.5 = \frac{y-100}{16} \Rightarrow 2.5(16) = y - 100 \Rightarrow y = 2.5(16) + 100 = 140$$

We need to get a score of 140 or higher to be considered a genius.

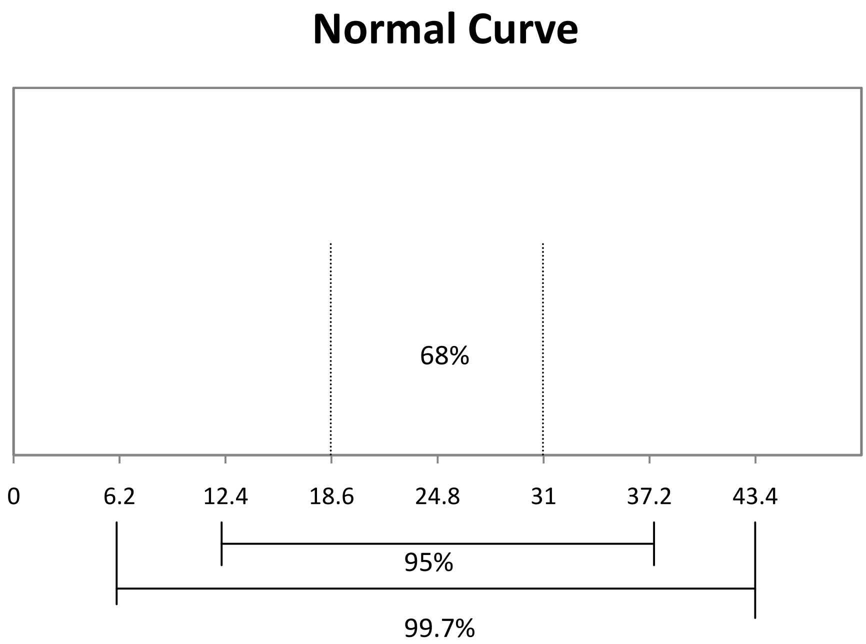
6.19] Cattle.

(a) $z = \frac{y-\mu}{\sigma} = \frac{1000-1152}{84} = -1.8095$. A 1,000 lbs steer is 1.81 standard deviations below the mean.

(b) The 1,000 lbs steer is more unusual. A 1,250 lbs steer has a *z*-score of $z = \frac{y-\mu}{\sigma} = \frac{1250-1152}{84} = 1.17$. The weight of 1,000 lbs has a *z*-score that is further away from zero.

6.27] Guzzlers?

(a) We have a N(24.8, 6.2) distribution.



(b) I'd expect the central 68% of autos to be between 18.6 and 31 mpg.

(c) About 16% of autos should get more than 31 mpg.

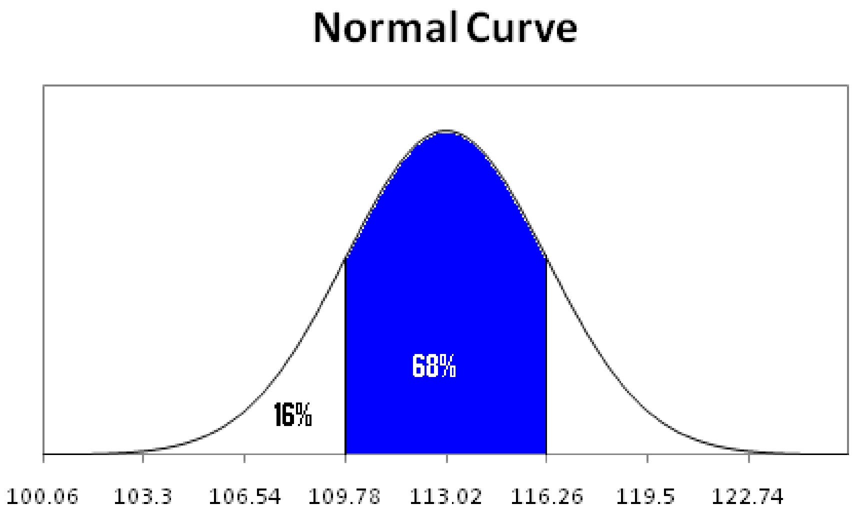
(d) About 13.5% of autos should get between 31 and 37.2 mpg.

(e) The worst 2.5% of autos should get less than about 12.4 mpg.

6.29] Small steer. Any weight more than 2 standard deviations below the mean, or less than 1152 – 2(84) = 984 pounds might be considered unusually low. We would expect to see a steer below 1152 – 3(84) = 900 very rarely.

6.31] Winter Olympics 2006 downhill.

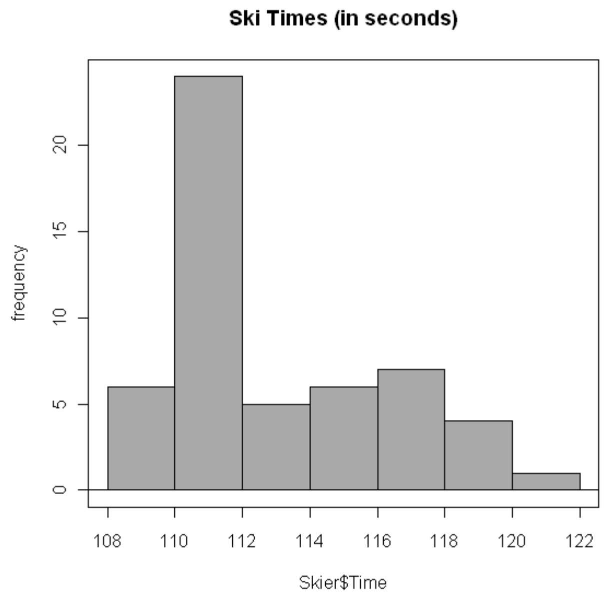
(a) First, note that 109.78 is one standard deviation below the mean. If the normal model is appropriate, we expect to have about 68% of times between 109.78 and 116.26 seconds. This means that 32% is left over, or 16% on both sides. We expect 16% of the skier times to be less than 109.78 seconds.



(b) The actual percentage of times below 109.78 is 2/53 = 0.0377 = 3.77%.

(c) The percentages don't agree. Most likely this is because we've assumed the data are normal when they really are not.

(d) The histogram is given below. Clearly, these data are not normal. The 68-95-99.7 rule shouldn't be used.

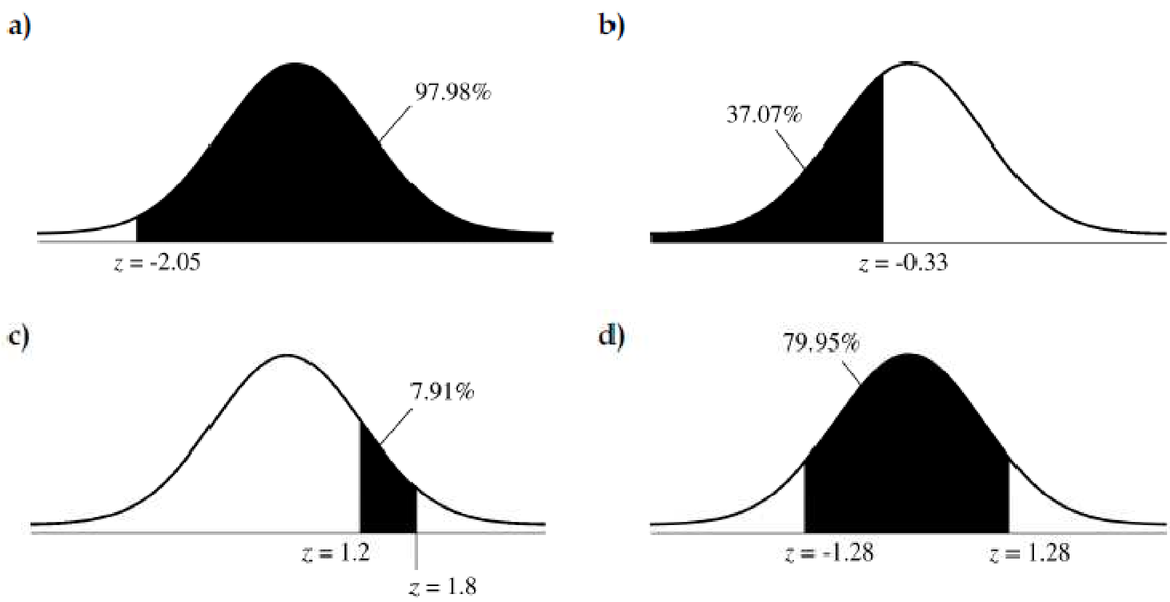


6.36] Check the model.

(a) We know that 95% of the observations from a Normal model fall within 2 standard deviations of the mean. That corresponds to 23.84 – 2(3.56) = 16.72 mph and 23.84 + 2(3.56) = 30.96 mph. These are also the 2.5 percentile and 97.5 percentiles, respectively.

b) The actual 2.5 percentile and 97.5 percentile are 16.638 and 30.976 mph, respectively. These are very close to the predicted values from the Normal model. I think the approximation from the Normal model is a good one.

6.40] Normal models, again.

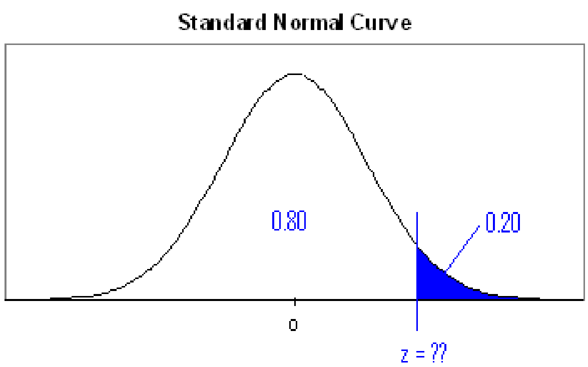


6.41] More Normal models. Find the cutoff for the following

(a) the highest 20%

The *z* with 80% to the left of it is z = 0.84 (from z-Table).

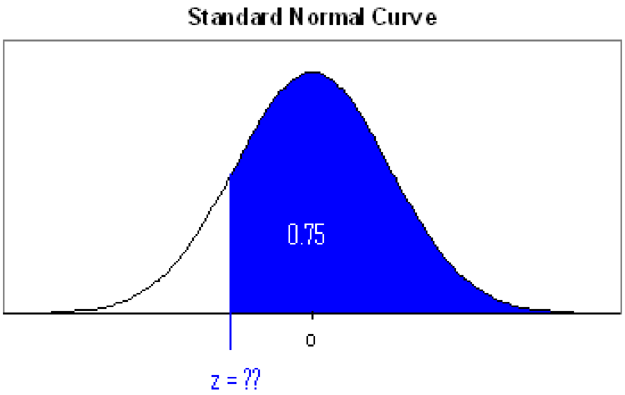
Alternatively, R commander gives z = 0.8416.



(b) the highest 75%

The z with 25% to the left of it is z = -0.67 (from z-Table).

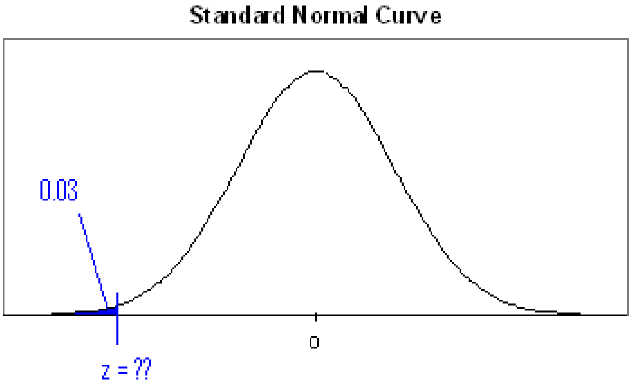
Alternatively, R commander gives z = 0.6745.



(c) the lowest 3%

The z with 3% to the left of it is z = -1.88 (from z-Table).

Alternatively, R commander gives z = -1.8808.

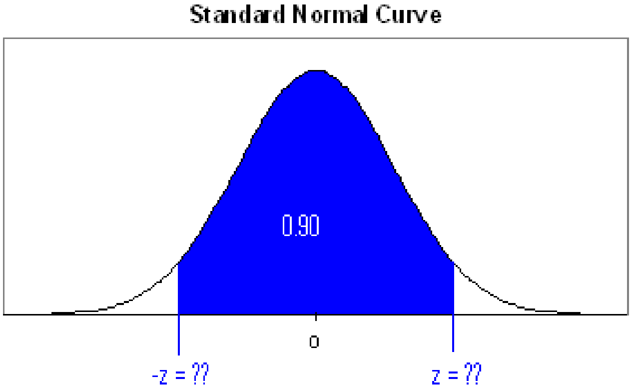


(d) the middle 90%

This is a little trickier. To get the middle 90%, we need a z value on the left with 5% below it, and on the right with 95% below it. The z with 5% to the left of it is z = -1.65 (from z-Table).

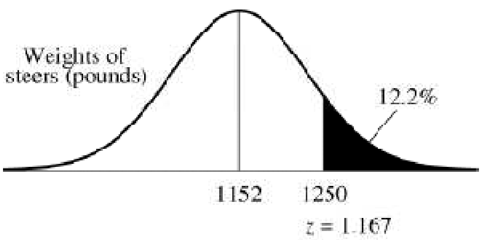
Alternatively, R commander gives z = -1.645.

We want $-1.645 < z < 1.645$.



6. 43] Normal cattle.

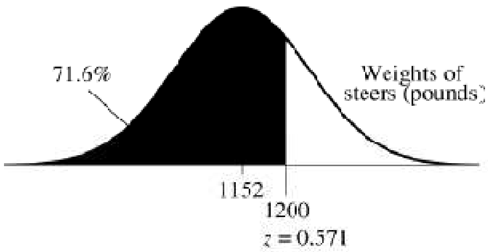
a)



$$z = \frac{y-\mu}{\sigma}$$
$$z = \frac{1250-1152}{84}$$
$$z \approx 1.167$$

According to the Normal model, 12.2% of steers are expected to weigh over 1250 pounds.

b)

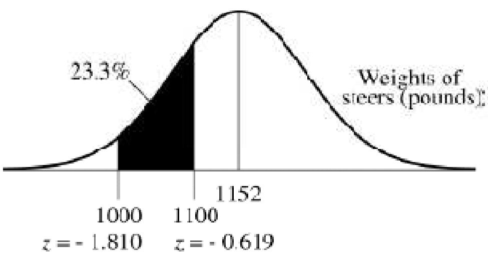


$$z = \frac{y-\mu}{\sigma}$$
$$z = \frac{1200-1152}{84}$$
$$z \approx 0.571$$

According to the Normal model, 71.6% of steers are expected to weigh under 1200 pounds.

c)

$$z = \frac{y-\mu}{\sigma} \qquad z = \frac{y-\mu}{\sigma}$$
$$z = \frac{1000-1152}{84} \qquad z = \frac{1100-1152}{84}$$
$$z \approx -1.810 \qquad z \approx -0.619$$

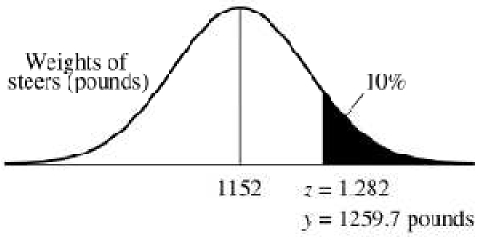


According to the Normal model, 23.3% of steers are expected to weigh between 1000 and 1100 pounds.

Note: You could have also used R commander here.

6.45] More cattle.

a)



$$z = \frac{y-\mu}{\sigma}$$

$$1.282 = \frac{y-1152}{84}$$

$$y \approx 1259.7$$

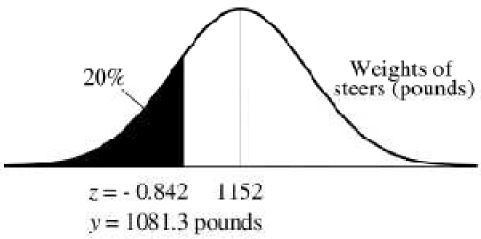
According to the Normal model, the highest 10% of steer weights are expected to be above approximately 1259.7 pounds.

b)

$$z = \frac{y-\mu}{\sigma}$$

$$-0.842 - \frac{y-1152}{84}$$

$$y \approx 1081.3$$



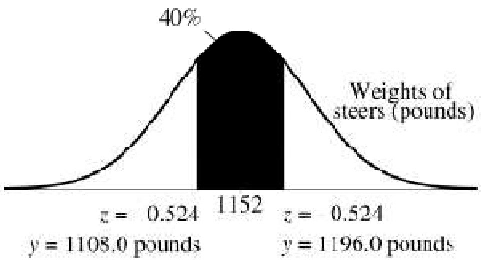
According to the Normal model, the lowest 20% of weights of steers are expected to be below approximately 1081.3 pounds.

c)

$$z - \frac{y-\mu}{\sigma}$$

$$-0.524 = \frac{y-1152}{84}$$

$$y \approx 1108.0$$



$$z - \frac{y-\mu}{\sigma}$$

$$0.524 = \frac{y-1152}{84}$$

$$y \approx 1196.0$$

According to the Normal model, the middle 40% of steer weights is expected to be between about 1108.0 pounds and 1196.0 pounds.

Note: You could also use R commander for this.