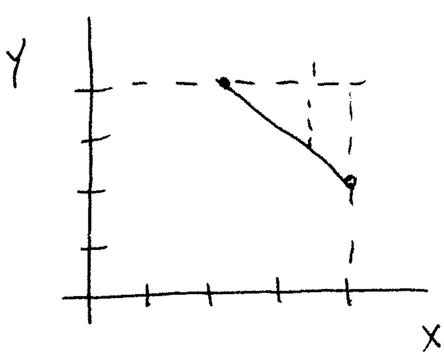
b. Show that X and Y are independent and have identical distributions (provide the marginal pdf they share).

$$f_X(x) = \int_0^4 \frac{xy}{64}\,dy = \frac{x}{64}\left(\tfrac{1}{2}y^2\right)\Big|_0^4 = \frac{x}{8}, \quad 0 \le x \le 4$$

$$f_Y(y) = \int_0^4 \frac{xy}{64}\,dx = \frac{y}{64}\left(\tfrac{1}{2}x^2\right)\Big|_0^4 = \frac{y}{8}, \quad 0 \le y \le 4$$

$$f_{XY} = \frac{xy}{64} = \frac{x}{8}\left(\frac{y}{8}\right) = f_X f_Y \iff X \perp Y$$

c. The two counties want to hire a single company for the repairs. One particular company will only handle combined jobs of at most 6 miles at a time for a given week before charging huge additional fees. Using a probabilistic argument (i.e. compute a meaningful probability), would you recommend the counties use this company for their repairs?



$$1 - P(X+Y > 6) = 1 - .34375 = .65625$$

$$P(X+Y > 6)$$

$$= \int_2^4 \int_{6-x}^4 \frac{xy}{64}\,dy\,dx$$

$$= \int_2^4 \frac{x}{64}\left(\tfrac{1}{2}y^2\right)\Big|_{6-x}^4 dx = \frac{1}{128}\int_2^4 x\left(16 - (6-x)^2\right)dx$$

$$= \frac{1}{128}\int_2^4 x\left(16 - 36 + 12x - x^2\right)dx = \frac{1}{128}\int_2^4 \left(-20x + 12x^2 - x^3\right)dx$$

$$= \frac{1}{128}\left(-10x^2 + 4x^3 - \tfrac{1}{4}x^4\right)\Big|_2^4 = \frac{1}{128}\left((-160 + 256 - 64) - (-40 + 32 - 4)\right)$$

$$= \frac{1}{128}\left(32 + 12\right) = \frac{44}{128} = .34375$$