Problem Set 7 for Math 30

Date Due: April 6, 2011

March 29, 2011

Chapter 16 Problems (Sections 16.1-16.4):
7, 9, 13 and Additional problems below

Additional:
1. Suppose the proportion θ of defective items in a large manufactured lot is unknown.
a. Suppose the prior distribution on θ is a uniform distribution on (0,1). When eight items are selected at random from the lot, it is found that exactly 3 of them are defective. Determine the posterior distribution of θ.
b. Now suppose the prior distribution on θ is Beta(2,200). If 100 items are selected at random and three are found to be defective, what is the posterior distribution of θ?

2. Suppose that the number of defects in a 1200 foot roll of magnetic recording tape has a Poisson distribution with mean θ, which is unknown.
a. Suppose the prior distribution on θ is a Gamma* distribution (3,1). Suppose five rolls (1200 feet each) are selected at random and the numbers of defects found are: 2,2,6,0, and 3. What is the posterior distribution of θ?
b. What is the Bayes estimate for θ? (Give notation and numerical value for this scenario.)

3. Suppose that the time in minutes required to serve a customer at a certain facility has an exponential* distribution with unknown parameter (θ). Suppose the prior distribution on θ is a Gamma* distribution with mean 2 and standard deviation 1.
a. If X is Gamma*(α, β), what is E(X)? What is V(X)?
b. What are the parameters of the Gamma* distribution used here as the prior?
c. If the average time required to serve a random sample of 20 customers is found to be 3.8 minutes, what is the posterior distribution of θ?
d. What is the Bayes estimate of θ? (Give notation and numerical value for this scenario.)

4. Suppose we are sampling from a normal distribution with unknown mean μ and precision τ. Suppose we sample n=11 observations, and obtain a sample mean of 7.2 and $s_{11}^2 = \sum (x_i - \bar{x})^2 = 20.3$. Suppose we assume a normal-gamma* prior for μ and τ with prior hyperparameters $\alpha_0 = 2$, $\beta_0 = 1$, $\mu_0 = 3.5$, and $\lambda_0 = 2$.
a. Find the posterior hyperparameters.
b. Find an interval that contains 95 percent of the posterior distribution of μ (i.e. a 95 percent CI for μ based on the posterior distribution).
c. Find an interval that contains 95 percent of the prior distribution of μ (i.e. a 95 percent CI for μ based on the prior distribution). Compare your intervals in b and c with a sentence or two.