Hmk 3 Solution for Math 29

Chapter 4

2ac $X = $ # women among 3 applicants

a.

$X =$	0	1	2	3
$P(X=x)$	.0454	.3182	.4773	.1591

$P(X=0) = \binom{7}{0}\binom{5}{3} / \binom{12}{3}$ etc.

$\binom{12}{3} = 220$

b. CDF $F(X) = \begin{cases} 0 & , x<0 \\ .0454 & , 0 \le x < 1 \\ .3636 & , 1 \le x < 2 \\ .8409 & , 2 \le x < 3 \\ 1 & , x \ge 3 \end{cases}$

6.b. P(at least 1 by cooking in 4)

= 1 − P(none by cooking)

= 1 − (P(not cooking))^4

$= 1 - \left(\frac{22300}{52006}\right)^4 = 1 - .0338 = .9662$

$\hookrightarrow .4288$

12 b.

x	0	1	2	4
$p(x)$	.20	.30	.35	.15

18 a. $\mu = 2.562$

$\sigma = \sqrt{1.958156} = 1.39934$

These are approx. values.

b. Median is 2. Mean is slightly larger and not integer valued.

27. $\mu = 5 \quad \sigma = .8$

a. $1 - \frac{1}{k^2} = .9 \Rightarrow k = 3.16$

$\mu \pm 3.16\sigma \Rightarrow (2.472, 7.528)$

b. Yes. Interval containing 90% is entirely below 8, so 8 is a rare event (max 10% in that tail)

32. Weird as it is : $P(X=i) = \frac{1}{7}$ for $i = 1, 2, \ldots, 7$ $\quad X = $ # keys.

so,

$\mu = \frac{1}{7}(1+2+\ldots+7) = 4$

$\sigma = \sqrt{\frac{1}{7}\left((1-4)^2 + \ldots + (7-4)^2\right)} = 2$

38.

X	0	1	2	3
p(x)	.140608	.389376	.359424	.110592

$P(X=0) = .52^3 \qquad P(X=3) = .48^3$

$P(X=1) = \binom{3}{1} .48 (.52)^2$

$P(X=2) = \binom{3}{2} .48^2 (.52)$

Can do μ, σ long hand or realize Bin (3, .48)

$\Rightarrow \mu = np = 1.44$

$\sigma = \sqrt{np(1-p)} = .8653$

46. a. $P(X=20) = \binom{40}{20}(.5)^{40}$

b. $P(X \le 15) = \sum_{i=0}^{15} \binom{40}{i}(.5)^{40}$

c. $P(X \ge 20) = \sum_{i=20}^{40} \binom{40}{i}(.5)^{40}$

d. No, but we are assuming they die independently.

e. $np = 40(.5) = 20$

f. $\sigma = \sqrt{np(1-p)} = \sqrt{10} = 3.16$

$\sigma^2 = 10$

50.

a. $P(X \geq 1) = 1 - P(X=0) = 1 - .9^{10} = .6513$

b. $E(\#\text{ sick}) = np = 1$

$E(\text{cost trt}) = 500(1) = 500$

$E(\underset{\text{rev}}{\text{total}}) = 500 + 10(80) = \underline{1300}$

54.

a. $P(X=2) = \binom{4}{2} .15^2 (.85)^2 = .0975375$

b. $P(X \leq 2) = \binom{4}{0} .15^0 (.85)^4 + \binom{4}{1} .15^1 (.85)^3$

$+ .0975375 = .98801875$ very likely

60. Note: This one is more annoying than I first thought.

3 judges $\perp$ vote $P(\overset{\text{vote}}{DP} \mid I) = .05$

$P(\text{get DP} \mid I) = \binom{3}{3} .05^3 (.95) + \binom{3}{2} .05^2 (.95)$ > either 2 or 3 judges vote for DP

$= .00725$

$P(\text{get DP } \underline{\text{and}}\ I) = .01(.00725) = .0000725$

$X = \#$ DP and Innocent $\sim Bin(100, .0000725)$

$\Rightarrow E(X) = np = .00725$ and

$\sigma(X) = \sqrt{np(1-p)} = .08514$

66. $p = .6$ Neg. Bin

a. $r=2$ $P(X \geq 3) = 1 - P(X<3)$

$= 1 - \left[\binom{1}{1} .6^2 (.4)^0 + \binom{2}{1} .6^2 (.4)^1 + \binom{3}{1} .6^2 (.4^2)\right]$

$= 1 - .8208 = .1792$

b. $r = 4$ $P(X \geq 3) = 1 - P(X<3)$

$= 1 - \left[\binom{3}{3} .6^3 (.4^0) + \binom{4}{3} .6^3 (.4^1) + \binom{5}{3} .6^3 (.4^2)\right]$

$= 1 - .9072 = .0928$

72. $r=4$ $p=.4$

$P(X=6) = \binom{9}{3} .4^4 (.6)^6$

$= .1003$

78. $r=5$ $P(\text{int}) = .6$

X = total she must ask

$Y = \#$ who say no

$X = Y+5$ $Y \sim NB(r=5, .6$

a. $P(X \leq 7) = P(Y \leq 2)$

$= \binom{6}{4} .6^5 (.4^2) + \binom{5}{4} .6^5 (.4)$

$+ \binom{4}{4} .6^5 (.4)^0$

$= .07776 + .15552$

$+ .186624$

$= .419904$

b. $E(Y) = \frac{5(.4)}{.6} = 3.33$

$V(Y) = \frac{5(.4)}{.36} = 5.55$

$\Rightarrow \underset{E(Y)+5}{E(X)} = 8.33$

$\underset{V(Y)}{V(X)} = 5.55$