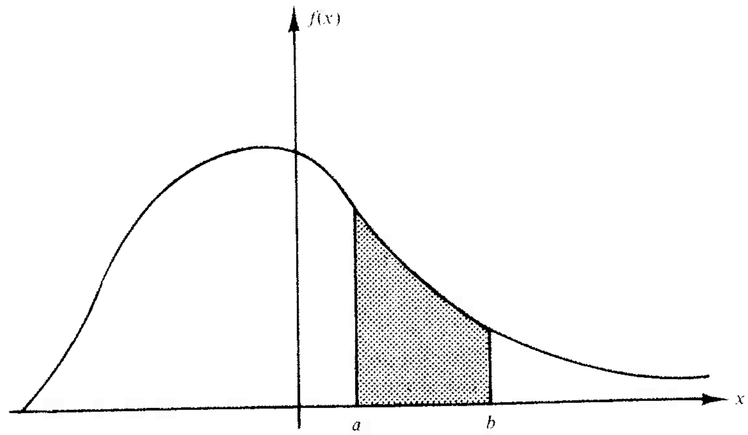

Figure 3.4 An example of a p.d.f.

Example 3.2.1 **Demands for Utilities.** In Example 3.1.5, we determined the distribution of the demand for water, X. From Fig. 3.2, we see that the smallest possible value of X is 4 and the largest is 200. For each interval $C = (c_0, c_1) \subset [4, 200]$, Eq. (3.1.1) says that

$$\Pr(c_0 < X \le c_1) = \frac{149(c_1 - c_0)}{29204} = \frac{c_1 - c_0}{196} = \int_{c_0}^{c_1} \frac{1}{196} dx.$$

Here we see that X has the p.d.f.

$$f(x) = \begin{cases} \dfrac{1}{196} & \text{if } 4 \le x \le 200, \\ 0 & \text{otherwise.} \end{cases}$$

◀

A typical p.d.f. is sketched in Fig. 3.4. In that figure, the total area under the curve must be 1, and the value of $\Pr(a < X < b)$ is equal to the area of the shaded region.

***NOTE:* Continuous Distributions Assign Probability 0 to Individual Values.** The integral in Eq. (3.2.1) also equals $\Pr(a < X < b)$ as well as $\Pr(a \le X \le b)$ and $\Pr(a \le X < b)$. Hence, it follows from the definition of continuous distributions that, if X has a continuous distribution, $\Pr(X = a) = 0$ for each number a. As we noted on page 17, the fact that $\Pr(X = a) = 0$ does not imply that $X = a$ is impossible. If it did, all values of X would be impossible and X couldn't assume any value. What happens is that the probability in the distribution of X is spread so thinly that we can only see it on sets like intervals. It is much the same as the fact that lines have 0 area in two-dimensions, but that does not mean that lines are not there. The two vertical lines indicated under the curve in Fig. 3.4 have 0 area, and this signifies that $\Pr(X = a) = \Pr(X = b) = 0$.

Nonuniqueness of the p.d.f.

If a random variable X has a continuous distribution, then $\Pr(X = x) = 0$ for every individual value x. Because of this property, the values of each p.d.f. can be changed at

Examp