Name:

Math 30 – Mathematical Statistics

Midterm 2 Practice Exam 1

Instructions:

1. Show all work. You may receive partial credit for partially completed problems.
2. You may use calculators and a one-sided sheet of reference notes. You may not use any other references or any texts, apart from the provided tables, and distribution sheet.
3. You may not discuss the exam with anyone but me.
4. Suggestion: Read all questions before beginning and complete the ones you know best first. Point values per problem are displayed below if that helps you allocate your time among problems, as well as shown after each part of the problem.
5. Good luck!

Problem	1	2	3	Total
Points Earned				
Possible Points				40

1. Suppose are you sampling from a Normal distribution with unknown mean and known variance equal to 4. In a Bayesian framework, suppose you assign a normal prior to the mean with mean 68 and variance 1. A random sample of n=10 observations results in a sample average of 69.5.

a. Circle all choices below that could appropriately fill in the blank.

In a situation like this, the prior is referred to as a(n) _____ prior.

informative uninformative improper proper conjugate complementary

b. State the posterior distribution of the mean.

c. Determine the Bayes estimator of the mean. How does that estimator compare to the Frequentist estimator - the sample average?

d. Determine a 95% Bayesian credible interval for the mean based on the posterior distribution, using equal tail probabilities.

2. Suppose you have a random sample of n exponentially distributed random variables, with unknown mean, β.

a. What result would allow you to identify a most powerful test of $H_0 : \beta = \beta_0$ vs. $H_A : \beta = \beta_A, \beta_A > \beta_0$?

b. Derive the most powerful test referred to in a. Make sure you highlight what statistic should be used in specifying the rejection region.

c. Is your test uniformly most powerful? Explain in one sentence.

d. Suppose you decide to test $H_0 : \beta = 20$ vs. $H_A : \beta > 20$, at a .05 level, and a random sample of 10 observations yields a sum of observations of 250. Would you be able to reject the null hypothesis? Explain.

3. A 1997 article on gun ownership surveyed 539 households and found that 133 of them owned at least one gun.

a. Is there significant evidence to suggest that more than 1/5 of households own at least one gun at a .05 significance level? Be sure to show your work, and use a rejection region approach.

b. Determine the p-value for your test in a.

c. Suppose that only 10 households could be surveyed, and we believe 20% of households own at least one gun (still versus alternative that is it > 20%). If you used a rejection region where RR={Y:Y>2}, where Y is the number of households with at least one gun owned in your sample of 10, what is:

i. the probability of the type I error

ii. the probability of a type II error if under the alternative the percentage is really 40%