Hmk 7 Solutions for Math 29

Chapter 6

6.4

a. X = Cause of Death where 1 = Accidents, 2 = CP, 3 = Assault, 4 = Other

Y = Sex / Gender where 1 = Female, 2 = Male (any coding works)

b. Using 4 decimals this is the jt. dist. Note this sums to .9999 so maybe 1 more dp is better

X / Y	1	2
1	.1270	.2108
2	.0510	.0580
3	.0389	.0482
4	.2054	.2606

$P(X=1, Y=1) = \frac{617}{4858} = .1270$ for example

c.

$Y=y$	1	2
$p(y)$	.4223	.5776

add the columns

again off by .0001

d.

$X=x$	1	2	3	4
$p(x)$	.3378	.1090	.0871	.4660

add up rows

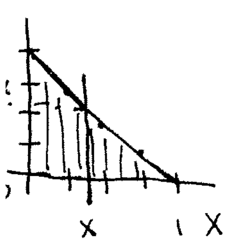
again off by .0001

6.14

a. $P(X < 1/2, Y > 1/4) = \int_0^{1/2}\int_{1/4}^{1} (x+y)\,dy\,dx = \int_0^{1/2} xy + \frac{y^2}{2}\Big|_{1/4}^{1} dx = \int_0^{1/2} \frac{3x}{4} + \frac{15}{32}\,dx$

$= \frac{3x^2}{8} + \frac{15x}{32}\Big|_0^{1/2} = \frac{21}{64} = .328125$

b. $P(X+Y \le 1) = \int_0^1\int_0^{1-x} (x+y)\,dy\,dx = \int_0^1 xy + \frac{y^2}{2}\Big|_0^{1-x} dx = \int_0^1 -\frac{x^2}{2} + \frac{1}{2}\,dx$



$= -\frac{x^3}{6} + \frac{x}{2}\Big|_0^1 = \frac{1}{2} - \frac{1}{6} = 1/3$

c. Marginals

$f_X(x) = \int_0^1 (x+y)\,dy = xy + \frac{y^2}{2}\Big|_0^1 = x + \frac{1}{2}$

Similarly $f_Y(y) = \int_0^1 (x+y)\,dx = y + 1/2$

d. $P(X > .75) = \int_{.75}^{1} x + \frac{1}{2}\,dx = \frac{x^2}{2} + \frac{x}{2}\Big|_{.75}^{1} = \frac{11}{32} = .34375$

use marginal

6.18 (skip e)

a. $P(CP \mid Male) = \frac{282}{2806} = .1005$

b. $P(Female \mid Homicide) = \frac{189}{423} = .4468$

c. CD given Male

$X=x$	1	2	3	4
$P(X=x)$	.3649	.1005	.0834	.4512

for example: $P(X=1 \mid Male) = \frac{1024}{2806} = .3649$

d. CD given Accidental

$Y=y$	1	2
$p(y)$	.3760	.6240

for example: $P(Y=1 \mid Acci) = \frac{617}{1641} = .37599 \approx .3760$

6.24

$$f(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{x+y}{y+1/2}$$ when $y=.25$ this is $\frac{x+.25}{.75} = \frac{4x+1}{3}$

$$P(X>.75 \mid Y=.25) = \int_{.75}^{1} \frac{4x+1}{3}\,dx = \frac{2x^2}{3} + \frac{x}{3}\Big|_{.75}^{1} = 3/8 = .375$$

6.26 $f(x,y) = e^{-x}$, $0 \le y \le x < \infty$, and 0, ow.

a. $P(X<2, Y>1)$

$$= \int_1^2 \int_1^x e^{-x}\,dy\,dx = \int_1^2 ye^{-x}\Big|_1^x dx = \int_1^2 xe^{-x} - e^{-x}\,dx$$

$$= \int_1^2 (x-1)e^{-x}\,dx \quad \begin{matrix} u=x-1 & du=dx \\ dv=e^{-x}dx & v=-e^{-x} \end{matrix} \quad = -(x-1)e^{-x}\Big|_1^2 + \int_1^2 e^{-x}\,dx$$

$$= -(x-1)e^{-x} - e^{-x}\Big|_1^2 = e^{-1} - 2e^{-2} = .0972$$

b. $$P(X \ge 2Y) = \int_0^\infty \int_0^{x/2} e^{-x}\,dy\,dx = \int_0^\infty ye^{-x}\Big|_0^{x/2} dx = \int_0^\infty \frac{x}{2}e^{-x}\,dx \quad \begin{matrix} u=\frac{1}{2}x & du= \\ dv=e^{-x}dx & v=-e^{-x} \end{matrix}$$

$$= \frac{1}{2}(-xe^{-x})\Big|_0^\infty + \frac{1}{2}\int_0^\infty e^{-x}\,dx = \lim_{x\to\infty} -\frac{x}{2}e^{-x} + \frac{1}{2}e^{-x} = \frac{1}{2}$$

c. $$f_X(x) = \int_0^x e^{-x}\,dy = ye^{-x}\Big|_0^x = xe^{-x}, \quad 0 \le x < \infty$$

d. $$f(x|y) = \frac{f(x,y)}{f_Y(y)}$$ Need $f_Y(y) = \int_y^\infty e^{-x}\,dx = -e^{-x}\Big|_y^\infty = e^{-y}$

So $$f(x|y) = \frac{e^{-x}}{e^{-y}} = e^{-x+y}, \quad y \le x < \infty$$ Note $x \ge y$ is part of the constraint.