Hmk 5 Solution for Math 27

7 total

4.140 from Sec. 4.11 $\quad a=1$ matrix $\quad b=2 \quad c=4$ → they need to get π

a.
$$\begin{array}{c|ccc} & A & B & C \\ \hline A & .3 & .3 & .4 \\ B & .4 & .4 & .2 \\ C & .5 & .3 & .2 \end{array}$$

b. $P(C \text{ in 2 stops} \mid \text{currently at } A)$

$$= P(AA)P(AC) + P(AB)P(BC) + P(AC)P(CC)$$

$$= .3(.4) + .3(.2) + .4(.2) = \underline{.26}$$

c. $\pi = \pi P \Rightarrow$

1 $\quad \pi_1 = .3\pi_1 + .4\pi_2 + .5\pi_3 \Rightarrow$

2 $\quad \pi_2 = .3\pi_1 + .4\pi_2 + .3\pi_3$

3 $\quad \pi_3 = .4\pi_1 + .2\pi_2 + .2\pi_3$

4 $\quad 1 = \pi_1 + \pi_2 + \pi_3$

1-2

$\pi_1 - \pi_2 = .2\pi_3$

$\pi_1 = .2\pi_3 + \pi_2$

4(2) - 3(3)

$4\pi_2 - 3\pi_3 = \pi_2 + .6\pi_3$

$\pi_2 = 1.2\pi_3 \Rightarrow \pi_1 = 1.4\pi_3$

Using 4,

$1.4\pi_3 + 1.2\pi_3 + \pi_3 = 1$

$3.6\pi_3 = 1$

$\pi_3 = \frac{5}{18} \quad \pi_1 = \frac{7}{18}$

$\pi_2 = \frac{6}{5}\cdot\frac{5}{18} = \frac{1}{3}$

$$\pi = \left[\, 7/18 \quad \underset{1/3}{\tfrac{6}{18}} \quad 5/18 \,\right]$$

Chapter 5

Max 45

.3 total

5.2 abc (lots of possible answers) 1 each

a. X=# stars - Discrete

other variables are possible

b. X= diastolic bp - Continuous

c. X= Tree DBH - Continuous (can argue D b/c rounding)

5.4 (no sketch) $f(x) = \begin{cases} cx(1-x), & 0 \le x \le 1 \\ 0, & \text{o.w.} \end{cases}$ total 8, 2 each part

a. $\int_0^1 cx(1-x)\,dx = \int_0^1 cx - cx^2\,dx = \frac{cx^2}{2} - \frac{cx^3}{3}\Big|_0^1 \Rightarrow c\left(\frac{1}{2} - \frac{1}{3}\right) = 1 \quad \underline{c = 6}$

b. $F(b) = \int_0^b 6x(1-x)\,dx = \frac{6x^2}{2} - \frac{6x^3}{3}\Big|_0^b = 3b^2 - 2b^3 \Rightarrow \underline{F(x)} = \begin{cases} 0, & x < 0 \\ 3x^2 - 2x^3, & x \in [0,1] \\ 1, & x > 1 \end{cases}$

c. exceed \$75 ⇒ x = .75 or higher

$\int_{.75}^1 6x(1-x)\,dx$ or $1 - F(.75) = 1 - \left(3(.75)^2 - 2(.75)^3\right) = 1 - .84375 = \underline{.15625}$

d. $P(X \ge .75 \mid X \ge .5) = \frac{1 - F(.75)}{1 - F(.5)} = \frac{.15625}{1 - \left(3(.5)^2 - 2(.5)^3\right)} = \frac{.15625}{.5} = \underline{.3125}$

5.8 bc X = # of minutes early 3 total

b. $P(X \ge 5) = 1 - P(X < 5) = 1 - \frac{20(5) - 5^2 - 40}{60} = 1 - \frac{35}{60} = \frac{25}{60} = \frac{5}{12} = .417$ 1

c. $f(x) = \begin{cases} \frac{x}{20}, & 0 \le x \le 4 \\ \frac{20 - 2x}{60}, & 4 \le x \le 10 \\ 0 & \text{o.w.} \end{cases}$ 2 check piecewise

5.12 7, 2 each $f(x) = 12x^2(1-x)$, $0 \le x \le 1$, 0, o.w.

$n=4$ $p = P(X \ge .5) = \int_{.5}^{1} 12x^2(1-x)\,dx = \frac{11}{16}$

$Y \sim Bin(4, p = 11/16)$

a. $P(Y=1) = \binom{4}{1}\left(\frac{11}{16}\right)\left(\frac{5}{16}\right)^3 = \underline{.0834}$

b. $P(Y \ge 1) = 1 - P(Y=0) = 1 - \binom{4}{0}\left(\frac{11}{16}\right)^0\left(\frac{5}{16}\right)^4$
$= 1 - .0095 = \underline{.9905}$

5.14 $f(x) = \frac{1}{2}$, $71 \le x \le 73$ 0, o.w. $\mu_X = \int_{71}^{73} \frac{x}{2}\,dx = \left.\frac{x^2}{4}\right|_{71}^{73} = \underline{72}$

2 each Uni(71,73)

$E(X^2) = \int_{71}^{73} \frac{x^2}{2}\,dx = \left.\frac{x^3}{6}\right|_{71}^{73} = 5184\,1/3$ $V(X) = E(X^2) - 72^2 = 1/3$

$\sigma_X = \sqrt{1/3} = \underline{.57735}$

5.16 2 each - 6 total

a. $E(X) = \int_0^\infty \frac{x}{5} e^{-x/5}\,dx = \underline{5}$ $E(X^2) = 50 = \int_0^\infty \frac{x^2}{5} e^{-x/5}\,dx \Rightarrow V(X) = \underline{25}$

b. Tcheby 75% $k=2$ $5 \pm 2(5) = \underline{(-5, 15)}$ $\xRightarrow{\text{can shift}}$ or $\underline{(0,15)}$ b/c can't be negative

c. $\int_t^\infty \frac{1}{5} e^{-x/5}\,dx = .125 \Rightarrow -e^{-t/5} + 1 = .875$
$-e^{-t/5} = -.125$
$-5\ln .125 = t$

OR

$\int_0^t \frac{1}{5} e^{-x/5}\,dy = .875$

$t = 10.397$

$\int_0^t \frac{1}{5} e^{-x/5}\,dx = .125 \Rightarrow -e^{-t/5} + 1 = .125$
$t = -5\ln .875$
$t = .6677$

is much smaller than Tcheby interval. $\underline{(.6677, 10.397)}$

5.20 4 - 2 each

a. $E(X) = \int_0^1 12x^3(1-x)\,dx = \underline{3/5}$ $E(X^2) = \int_0^1 12x^4(1-x)\,dx = 2/5 \Rightarrow V(X) = 1/25$
$\sigma(X) = \underline{1/5}$

b. $Y = 200 - 200X \Rightarrow E(Y) = 200 - 200E(X) = \underline{80}$
$V(Y) = 200^2 V(X) = 200^2(1/25) = \underline{1600}$

Additional. a = 1 matrix b = 3 ← 2 steps · work initial prob. c. 4 π. 8 total

a.

$$\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 0 & .2 & .8 \\ .6 & 0 & .4 \\ .5 & .5 & 0 \end{bmatrix}$$

b. time n like time "0"

$p^{(0)} = [1/3, 1/3, 1/3]$

want $p^{(2)} = p^{(0)}P^2$

$$P^2 = \begin{bmatrix} .52 & .40 & .08 \\ .20 & .32 & .48 \\ .30 & .10 & .60 \end{bmatrix}$$

$p^{(2)} = \left[\frac{1}{3}(1.02) \quad \frac{1}{3}(.82) \quad \frac{1}{3}(1.16)\right] = \underline{[.34 \quad .2733 \quad .3867]} \Rightarrow$ Boy C most likely to have @ time n+2

c. $\pi = \pi P$

$\pi_1 = .6\pi_2 + .5\pi_3$ (1)

$\pi_2 = .2\pi_1 + .5\pi_3$ (2)

$\pi_3 = .8\pi_1 + .4\pi_2$ (3)

$\pi_1 + \pi_2 + \pi_3 = 1$ (4)

(2) - (1)

$\pi_2 - \pi_1 = .2\pi_1 - .6\pi_2$

$\pi_2 = 3/4\,\pi_1$

(3) - 4(2)

$3\pi_3 = 4.4\pi_2$

$\pi_3 = \frac{22}{15}\pi_2 = \frac{11}{10}\pi_1$

Using (4)

$\pi_1 + \frac{3}{4}\pi_1 + \frac{11}{10}\pi_1 = 1$

$\pi_1 = \frac{20}{57}$

$\pi_2 = \frac{15}{57}$

$\pi_3 = \frac{22}{57}$

$\underline{\left[\frac{20}{57} \quad \frac{15}{57} \quad \frac{22}{57}\right]} = \pi$