## Problem Set 3 for Math 30

Date Due: Feb. 17, 2011

February 11, 2011

Sections 9.2-9.4 Chapter 9 Problems: 3, 6, 15, 17, 25, 26, 31, 32, 39, 42, 46, 51, 52

Additional:

Let  $X_1, X_2, ..., X_n$  be a random sample from a  $N(\mu, \sigma)$  distribution, where  $\mu$  and  $\sigma$  are both unknown, but are constant (and exist). Recall that  $X = (n-1)s^2/\sigma^2$  has a chi-squared (n-1) distribution. The steps below will lead you to prove consistency of  $s^2$  for  $\sigma^2$  directly (without using the trick we did in class).

- a. What is the distribution of X in terms of the Gamma distribution?
- b. Let  $Y = \frac{n-1}{n}s^2$ . What is the distribution of Y?
- c. Would you be able to use Y as a pivot for  $\sigma^2$ ? Explain in one sentence.
- d. Find  $Var(s^2)$  using the distribution you found in b.
- e. Show consistency of  $s^2$  for  $\sigma^2$  without using the trick we did breaking up the summation.

Notes: 31 and 32 are practice with law of large numbers. 51 and 52 use a "trick" to show the sufficiency.

Other notes: Hardest problems on the homework are: 26, 39 (it says to show sufficiency the long way), 51, and 52 (if you don't get the trick for the last two).